Lecture 1: Reproducing kernel Hilbert spaces and interpolation.

Speaker: Nikolaos Chalmoukis, Universitá di Bologna.

Date and time: 17.11.2021, 18.00 (Moscow)

Abstract. Interpolation problems by analytic functions is a subject of more than a century old that is rich of interesting and deep results but also continues to stimulate new research. Interpolation has come a long way since the fundamental papers of Pick [5] and Nevanlinna [3, 4], but the spirit of the problems is invariant. One is given a subset of analytic functions in the unit disc $\varepsilon \subset \mathcal{O}(\mathbb{D})$, a sequence (finite of infinite) $\{z_i\} \subset \mathbb{D}$ of interpolating nodes and a *target space* $\mathcal{X}$, i.e. a set of sequences to be interpolated. The problem is then to determine whether or not for any data $\{w_i\} \in \mathcal{X}$ there exists a function $f \in \varepsilon$ such that

$$f(z_i) = w_i, \quad \forall i.$$

In this lecture we will introduce the notion of a reproducing kernel Hilbert space and we will formulate an interpolation problem on this setting. We then shall use the language of base theory of Hilbert spaces in order to reformulate the problem of interpolation.

Lecture 2: Complete Nevanlinna Pick spaces.

Speaker: Nikolaos Chalmoukis, Universitá di Bologna.

Date and time: 19.11.2021, 18.00 (Moscow)

Abstract. Carleson's [2] work is a milestone in the theory of interpolation. Carleson gave a characterization of all sequences $\mathcal{Z} = \{z_i\} \subset \mathbb{D}$ such that for any bounded data $\{w_i\} \subset \ell^\infty$ there exists a bounded analytic function $f \in H^\infty(\mathbb{D})$ such that $f(z_i) = w_i,\ i = 1, 2, \ldots$

Carleson's result had a profound impact in the function theory of the Banach algebra $H^\infty(\mathbb{D})$ and in particular it lies in the heart of the proof of the Corona theorem [1] for $H^\infty(\mathbb{D})$. Later work of Shapiro and Shields [6] completed the picture by considering a related interpolation problem for the Hardy space $H^2(\mathbb{D})$. A sequence $\mathcal{Z} = \{z_i\}$ is called interpolating for $H^2(\mathbb{D})$ if for any $\alpha \in \ell^2$ there exists $f \in H^2(\mathbb{D})$ such that

$$f(z_i) = \alpha_i(1?|z_i|^2)^{-\frac{1}{2}}, \quad i = 1, 2, \ldots$$

Shapiro and Shields [6] showed that the two notions coincide; a sequence is interpolating for $H^2(\mathbb{D})$ if and only if it is interpolating for $H^\infty(\mathbb{D})$. We will present a

generalization of Carleson's Theorem which holds for all spaces with the so called complete Nevanlinna Pick property. Surprisingly, universally interpolating sequence are still characterized by a Carleson measure condition and a weak separation condition in this much more general setting. There exist two different proofs of this fact, one of which is based on the solution of the Kadison - Singer problem and is quite simple.

Lecture 3: The Dirichlet space and simple interpolation.

Speaker: Nikolaos Chalmoukis, Universitá di Bologna.

Date and time: 24.11.2021, 18.00 (Moscow)

Abstract. In this lecture we will discuss simply interpolating sequences. In contrast to the universally interpolating sequences, there exists no characterization of such sequences which holds for all complete Nevanlinna Pick spaces. As a matter of fact, the only space on which genuine simply interpolating sequences have been shown to exist (i.e. which are not universally interpolating) is the Dirichlet space in the unit disc. This is defined as the Hilbert space of analytic functions with finite Dirichlet integral;

$$\int_{\mathbb{D}} |f'|^2 \, d\, A < +\infty.$$

We shall present a characterization of simply interpolating sequences in the Dirichlet space. The same characterization is conjectured to hold in all complete Nevanlinna Pick spaces but the problem remains open despite recent progress.

Lecture 4: Variations on a theme: Random interpolating sequences.

Speaker: Nikolaos Chalmoukis, Universitá di Bologna.

Date and time: 24.11.2021, 18.00 (Moscow)

Abstract. Finally, in the last part of the course we are going to discuss some variants of the classical interpolation problem, such as random interpolation. This is a field where numerous questions remain open. In the context of the standard weighted Dirichlet spaces in the unit disc we will discuss a random version of the

classical interpolation problem. In particular we consider Steinhaus random variables and we derive the Kolmogorov 0-1 law for universal interpolation, weak separation and the Carleson measure condition. It turns out that in the probabilistic setting there exist simple characterizations of almost surely interpolating sequences. In particular universally and simply interpolating sequences coincide almost surely.

References

[1] L. Carleson. On a class of meromorphic functions and its associated exceptional sets. Appelberg, 1950.

[2] L. Carleson. An interpolation problem for bounded analytic functions. Amer. J. Math., 80(4):921, oct 1958.

[3] R. Nevanlinna. ?ber beschr?nkte funktionen die in gegebenen punkten vorgeschriebene werte annehmen. Ann. Acad. Sci. Fenn. Ser. A, 13:1–71, 1919.

[4] R. Nevanlinna. ?ber berschr?nkte Funktionen. Ann. Acad. Sci. Fenn. Ser. A, 32(7), 1929.

[5] G. Pick. ?ber die beschr?nkungen analytischer funktionen, welche durch vorgegebene funktionswerte bewirkt werden. Math. Ann., 77(1):7–23, March 1915.

[6] H. S. Shapiro and A. L. Shields. On some interpolation problems for analytic functions. Amer. J. Math., 83(3):513–532, 1961.